UNDERSTANDING EPISTEMOLOGICAL DIVERSITIES IN MATHEMATICS CLASS

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ABSTRACT

Recently, the notion of community has been increasingly popular in theoretical discourse of mathematics education and become a basic unit for analysis of classroom interaction. In this context, as part of ethnography in university level mathematics classes in the US investigating social transformation in mathematics education, this paper intends:

(1) to examine the notion of “mathematics classroom as community” as a place of learning and
(2) to identify some educational implications for teaching mathematics.

The data were collected through classroom observation and interviews. The analysis focused on comparing notions of mathematics shared among different groups of mathematicians, i.e., novices and old-timers. Through the comparison, I found that there are not only differences but also similarities in their understanding of what mathematics is and that they are intricately related to one another to constitute a practice of mathematics as a whole. Such complexity leads to a conclusion that mathematics class as a community is neither closed nor self-contained. It is interacting with outside communities. Each participant in mathematics class is representative of a community that s/he is committed to and his/her way of thinking mathematically reflects the epistemological standpoint of the community. This suggests that mathematics class is a community where diverse communal epistemological standpoints are renegotiated and that it creates its own unique mathematical culture through the negotiation among its participants. From this point of view, the understanding of the epistemological diversity is important for mathematics teachers to support successful learning.

KEY WORDS: Mathematics Class, Community of Practice, Ethnography, Indigenous Epistemology, Epistemological Diversity
1. Introduction

Recently, the notion of community has been increasingly popular in theoretical discourse of mathematics education and become a basic unit for analysis of classroom interaction to provide deep insight into teaching and learning mathematics in school (Cobb & Bauersfeld, 1995; Lampert & Blunk, 1998; Voigt, Seeger, & Waschescio, 1998). Compared to traditional educational research regarding mathematics class as a semi-laboratory where value-neutral skills are transmitted from a teacher to students in a vacuum of meaning, sociocultural approaches to mathematics education have revealed ways of speaking, seeing, thinking mathematics particular to school mathematics class. In the perspective, mathematics class is a community where participants negotiate their mathematical meanings and ways of doing mathematics to create its own mathematical culture through daily practice of mathematics (Cobb & Bauersfeld, 1995; Cobb, Wood, & Yackel, 1996; Ju, 2000; 2001; Lampert & Blunk, 1998; Voigt, 1985; Voigt, Seeger, & Waschescio, 1998).

In this context, this paper is to present the result of a comparative analysis of understanding of “what is mathematics” between different groups of mathematicians, that is, old-timers and novice mathematicians in a university mathematics department in order to reveal the intricateness of the notion “a community of practice” as a place for teaching and learning. Some educational implications for teaching mathematics will be presented based on the findings from the analysis.

2. Research Setting: Mathematics Classes in a University

This is part of the ethnographic research in university level mathematics classes in the US during 1998-1999 academic year. In the research, the author had collaborated with an experience mathematics teacher who had taught mathematics at the university for nearly thirty years. The data were collected through participatory observation of the professor’s mathematics classes at three different levels: an introductory calculus class, an advanced undergraduate mathematics class, and a graduate mathematics class. Some sessions were video-recorded for further discourse analysis. In addition to classroom observation, forty people were interviewed. The purpose of interview was to learn about notions of mathematics shared in the mathematics department. The interviewees were selected to reflect the diversities in cultural backgrounds and in the level of mathematical expertise in the mathematics department. The interviews were audiotaped for later detailed analysis.

As mentioned, the analysis of this paper will focus on comparing notions of mathematics shared by two different groups of mathematicians, i.e., novices and old-timers. In the

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1 In interview, some interviewees brought up the point that mathematics is “what they do daily”. Based on that, the term “a mathematician” will be used rather inclusively, that is, people “doing” mathematics instead of referring to a professional with a degree. However, it is important to note that the term always carried cultural connotation of legitimacy, as a graduate student pointed out, “I have something new to offer, so a new insight, [to the community of mathematics]. Then I would consider myself a mathematician, a research mathematician.”

2 Although the mathematicians in the department did not unanimously agree upon the idea of that time or the advance in status does exactly predict the development of some essential qualities of a mathematician such as creativity, it was often observed that “time” was taken as one of the most prevalent dimensions in defining a position of a mathematician in the department. Also, it was possible to distinguish kinds of practice of mathematics in terms of the length of engagement with mathematics.
analysis, language is taken as the unit of analysis based on the assumption that use of language reflects the cultural organization of lived experience in a society (Gumperz & Levinson, 1996; Hill & Mannheim, 1992; Hymes, 1974; Whorf, 1956):

“Facets of cultural values and beliefs, social institutions and forms, roles and personalities, history and ecology of a community may have to be examined in their bearing on communicative events and patterns” (Hymes, 1974, p.4).

3. Different Kinds of Practice of Mathematics

It is well known that novices are more likely to regard mathematics as a product, that is, fixed body of skills independent of human beings, while old-timers think of mathematics as a process of problem solving. This kind of tendency was confirmed by the data collected in the mathematics department. The beginning mathematics students regarded mathematics as a logically structured fixed network of mathematical products such as mathematical laws and rules. On the contrary, the old-timers considered mathematics largely as a process, in other words, “what people do” and refused the notion of mathematics as a fixed structure of impersonal knowledge. For instance, in the advanced mathematics classes taught by the professor, there were several students who retook the classes. They already had taken the courses but with another lecturer, and revisited the course then. If mathematics were fixed, immutable, and impersonal knowledge, for what did they come back to learn “the same things” over again?

Interviewer: Why do you take the class?

Interviewee: I already took the class last semester with another professor. But I know..I knew that the professor is teaching it again this spring. And I thought that because very professor has a different point of view..it is just like humanity. Everybody has a different point of view and different experience that they bring to any class. And I knew the professor has a great expertise in the subject. So I knew that he would invaluably have many insights and he would have wealth of experience to share with us. So although I was taking the class before, I want to sit in just to hear his point of view.

As mentioned, the old-timers of the mathematics department regard mathematics as their daily practice as a whole, that is, “what we do” rather a definite structure of mathematical propositions. In their practice, mathematicians are personally engaged with mathematics and the structure of mathematical knowledge is continually evolving through the practitioners’ creative imagination. The evolution is deeply related to their personal mathematical experience which provides unique meaning to logical connections in a mathematical structure. In this regard, as the interviewee suggested in the above transcript, mathematics that the professor actually taught in his classes was significantly different from mathematics given in “a book” or in the official descriptions of the courses given in the General Catalog of the university. Basically, he covered the contents of the courses. But he did not simply regurgitate definitions and theorems as given in a book. The majority of class was allotted for interpretations in order to give a student a perspective on a mathematical product under discussion: for instance, what does a concept or a theorem tell, how is it connected to a

Thus, the categories of “novices” and “old-timers” will be used for the purpose of the comparative analysis in this paper.
broader structure of mathematics” what is its implication for the future development of the subject, and so forth.

This kind of knowledge, that is, “a mathematical point of view”, is rarely found in “a book”. Rather, it manifests itself through daily practice of mathematics including mathematical communication among colleagues. In this regard, for old-timers, mathematics includes not only a set of final statements but also evolving intersubjective meaning. And it is this latter kind of mathematics which old-timers emphasize in their teaching and learning.

So far, the comparative analysis has highlighted differing understanding of mathematics by different groups of mathematicians. However, this does not suggest that their practice of mathematics follow an either-or scheme. Indeed, it is important to note that the dichotomy “a product vs. a process” provides only a reductive model to understand mathematical practice of each group. In general, it is considered that a deeper scrutiny into daily practice of mathematics will disclose a complicated picture behind the dichotomy and provide a more meaningful understanding of mathematics classroom as a place of learning, which following further analysis purports.

Although the old-timers considered mathematics as a process of developing a perspective on the world, they never underestimated the importance of the aspect of mathematics as a product in their practice of mathematics. For instance, in mathematics class, the old-timers taught specific definitions, theorems, algorithms, computing procedures, and so on. Advanced mathematics students tried to memorize definitions, theorems, algorithms, and mathematical proofs as beginning mathematics students did. The old-timers may be doing these kind of technical things for practical purposes such as preparing for exams. Thus, despite the differing understanding of what mathematics is, it turns out that practice of old-timers is also concerned with mathematics as a product in a certain way. Based on this similarity, the further analysis is to reveal difference in practice of mathematics at a more fundamental level by arguing that mathematicians apply different meanings, or more generally speaking, cultural epistemological standpoint, to their practice of mathematics. I will elaborate this idea by showing different meaning imposed on the shared interest in mathematics as a product.

As noted, although the old-timers values creativity over technical perfection in their practice of mathematics, technical development cannot be separated from developing mathematical creativity:

“Somebody comes up with some ideas and the idea itself somehow brings some form already and another whole set of questions.”

As the interviewee describes in the above, a mathematician begins his/her creative inquiry with a question based on a rather vague idea. Through creative mathematical investigation, the question evolves into “a form”, that is, a mathematical product such as a theorem. In turn, the form initiates another mathematical inquiry to lead to another mathematical discovery, and so forth. This recursive process suggests that mathematical creativity is the origin of the formal objective mathematical proposition. In addition, the process implicitly assumes that formal mathematical knowledge is a language to encode mathematical creativity of a mathematician. In other words, mathematical creativity is firmly grounded on factual knowledgeability in mathematics.

Furthermore, the process does not happen in a vacuum of meaning but is shaped by the invisible hand, “the culture of the mathematics community”: 
“There are a number of possible combinations of axioms for example. It’s infinite. And if we make some random deduction and publish a paper, that’s silly… There is so to speak a sixth sense that tells you whether something is significant or not.”

A mathematical discovery is usually the object of examination in the community of mathematics. The members of the mathematics community scrutinize its logical perfectness and meaningfulness of a mathematical discovery with respect to the current mathematical structure, and more importantly, its creativity and productivity for future development of mathematics, in other words, judging whether the research has something new to offer to the mathematics community. In this regard, mathematical products such as computational skills and theorems are the culmination of the cultural norm of doing mathematics in the sense that they have acquired its social status as a consequence of on-going social review based on the social norms. Thus, not only a step by step guideline to solve a particular problem, a mathematician learns communal voice behind technicality such as what it is concerned with, what is the idea behind it, what and how it suggests doing to further a mathematical idea, and how to put a creative idea in a culturally meaningful way.

Therefore, while techniques are usually a terminal point for a novice mathematician, they become departing point for a future practice of mathematics for old-timers. Furthermore, techniques are cultural language to materialize creative vision for future practice of mathematics as communicating cultural norms of mathematics community to a practitioner. Through the process of communication, a mathematician becomes socially transformed according to the cultural norms of mathematics community and the communication becomes intense as a mathematician grasps the spirit of communal practice through participation. Put differently, when old-timers deal with mathematics as a product, their practice follows the cultural norms of doing mathematics shared in the mathematics community. Therefore, this tells that it is the notion of cultural norms, more generally communal epistemological standpoint, which will be defined in the next, what produces the differences between mathematical practice of old-timers and of novices at the surface level.

4. Communal Epistemological Standpoints in Practice of Mathematics

Indigenous epistemology is concerned with both the theory of knowledge and theorizing knowledge, including the nature, sources, frameworks, and limits of knowledge sociohistorically developed in a cultural group. Specifically, indigenous epistemology refers to a cultural group’s ways of thinking and of creating, reformulating, and theorizing about knowledge such as who can be a knower, what can be known, what constitutes knowledge, sources of evidence for constructing knowledge, what constitutes truth, how truth is to be verified, how evidence becomes truth, how valid inferences are to be drawn, the role of belief in evidence, and related issues via traditional discourses and media of communication, anchoring the truth of the discourse in culture (Gegeo & Watson-Gegeo, 2001). Since an epistemological system is socially constructed and informed through sociopolitical, economic, and historical context and processes, it is a community that is a primary epistemological agent and that provide a basis for theorizing knowledge (Alcoff & Potter, 1993; Gegeo & Watson-Gegeo, 2001).

In the above analysis, different ways of doing, specifically different understanding of what is mathematics has been compared. In particular, the analysis focused on the old-timers’
practice of mathematics -- not only how to construct logical mathematical reasoning but their understanding of legitimate conduct of mathematics, in general --, and showed that their practice is constituted by sociocultural values and norms of doing mathematics, that is defined as an indigenous epistemology.

For last several decades, in diverse disciplines, research has shown that knowledge is socioculturally constructed and mathematics is not an exception (See Berger & Luckmann, 1966; Bloor, 1991; Joseph, 1994; Restivo, 1994). Based on that, the notion of “ethnomathematics” has been developed to contribute to the awareness of sociocultural aspect in mathematical reasoning, especially “culture” in mathematics learning (Ascher & Ascher, 1997; D’Ambrosio, 1985; Powell & Frankenstein, 1997). However, it is necessary to point out that early sociocultural studies were based on a superficial interpretation of “culture” as a definite repertoire of behavioral patterns and as a result, theory of multicultural education tended to reductively treat culture as “colorful customs of other people” (Watson-Gegeo, 2001). In this regard, the notion of indigenous epistemology make it possible to investigate culture of mathematics classroom at a deeper level to provide stronger theoretical perspectives to improve teaching and learning mathematics.

For the purpose, I consider that it is important to extend the notion of indigenous epistemology argument in order to explain the mathematical practice of novice mathematicians? Put differently, novices as well as old-timers should be considered as communal being instead of as isolated atomic individuals but possessing communal epistemological point of view different from that presented by old-timers. In their practice of mathematics, novices apply the indigenous epistemological standpoints of the community that they have been committed to rather than that of the mathematics community in which they just begin to participate.

For example, in daily conversation with students in introductory mathematics classes, it was often observed that novices’ discourse about mathematics was organized around the notion of “economy” such as “time management” -- e.g., spend “less” time to get “more” grades --, “exchange” -- e.g., need A to apply to medical school --, “possession” -- “I know everything about derivative” --, and so forth. These ways of speaking about mathematics reflect the epistemological position shared in communities outside of the mathematics community. Due to the lack of engagement, outsiders rarely have opportunity to develop a sophisticated understanding about what mathematics is than people who practice mathematics daily. In daily practice, mathematics keeps emerging every moment of engagement by a mathematician. However, mathematics has a smooth and perfect outlook only at a distance. Moreover, most often, mathematics is presented as completed knowledge in school and at home by adults through their expectation. And in modern society, mathematics is regarded as knowledge with most potential for future production (Stehr, 1994; Popkewitz, 1991)

Therefore, it can be said that novice mathematicians’ practice of mathematics is deeply situated within their understanding of mathematics shaped through their lived experience in the communities they have been socialized into such as home, high school, a capitalistic society. In this respect, it is interesting to point that this kind of pattern could be found among some students in the advanced mathematics class. In the advanced undergraduate mathematics class, there were several students who came from outside departments such as school of engineering. Compared to the students in the introductory mathematics class, they had participated in more mathematics classes and had a strong mathematical background. Despite the difference in mathematical expertise, they were similar to the beginning
mathematics students in the sense that they evaluated the practice of mathematics in the class from the viewpoint of the community that they have been committed to. For instance, a Ph.D. student from the civil engineering department compared mathematics to civil engineering to criticize its technicality in the interview:

“So for mathematicians, he is probably surprised if you cross the street to engineer department and try to learn something about continuum mechanics. They are very different. Because we are not so rigorous, we can do things faster for example. I think that it is very interesting. Like you say your mathematics is less rigorous and fast. But for example, the Bay Bridge, that’s constructed by an engineer.”

The interviewee came to the mathematics class because he thought that mathematics would provide a valuable insight into physical phenomena. However, he did not agree to the way of doing mathematics in the mathematics department because that does not match his communal epistemological norms concerning what is a valuable kind of knowledge. On the contrary, such kind of mathematical practice gives joy and meaning to people in the mathematics department, as a graduate student of the mathematics department says in the following transcript:

“You’re absolutely right. It is difficult. But at the same time, that’s exactly what makes fun that when you finally do understand something. It is really wonderful. And very frequently it turns out to be quite beautiful, the answer. And then those are quite of motivation, I think.”

As a mathematics major student describes in the above, the technicality and the abstractness of mathematics causes difficulty in their practice, but ultimately brings meanings. Old-timers have developed “enlightened eyes” to see the beauty of the mathematical structure they have created historically. But the meaning and the beauty cannot be grasped by people who do not share the epistemological norm of the mathematics community.

**5. Over the Wall: Is a Difference a Sign of Deficit?**

Sociocultural approaches to mathematical problem solving have revealed sociocultural nature of mathematical reasoning and the research findings have been related to development of new theoretical perspectives on how to improve teaching and learning in school mathematics classroom (See Lave, 1988; Nunes & et als., 1993). However, cumulating sociocultural research findings imply that cultural influence must be much more fundamental. From the perspective, this study intended to investigate “deep culture” of mathematics class.

For the purpose, this paper presented the result of a comparative analysis of practice of mathematics in a mathematics department, in particularly, focusing on shared notion of what mathematics is. Based on the comparison, this paper introduces the notion of indigenous epistemology and argues that mathematics class consists of diverse kinds of participants in the sense that they bring diverse kinds of communal epistemological standpoints to the class. Moreover, each participant practices mathematics according to an indigenous epistemological standpoint of a community that s/he has been committed to.

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3 By “deep culture”, Watson-Gegeo refers to a deeper level of thinking and understanding, in other words, “below the surface level of behavior and the linguistic level of morphology and syntax, a deep set of propositions and images that shapes perception, information processing, and the assignment of values” (Watson-Gegeo, 2001, p. 10).
This suggests that a mathematics classroom as a practice community is neither closed nor self-contained. It is deeply related to outside communities in the sense that each participant represents the indigenous epistemological standpoint of the community that s/he is committed to. Through interaction, a student begins to grasp different ways of doing mathematics, different epistemological style and begins to change. When considering that epistemology is not restricted to cognition in a narrow sense, such change is fundamental. That is, it is negotiation of worldview of a learner. And through the change, the epistemological standpoint of the community also becomes transformed and in fact, it is the product of such historical contingency created by the interacting participants instead of an immutable transcendental given.

As refuting Eurocentrism in mathematics education, sociocultural studies of mathematics have provided theoretical basis for understanding difference to improve teaching and learning mathematics in school. In this aspect, it is important to point out that it is epistemological difference and confliction due to such difference that initiate learning and make the impact of learning more fundamental. And this provides a new perspective on difference, which has been seen as deficit in traditional mathematics classrooms (Voigt, 1998). When considering that a mathematics classroom is a practice community with a particular epistemological standpoint and that indigenous epistemology is much broader than a set of mathematical concepts and skills given in a curriculum, it can be said that misunderstanding of epistemological difference affects teaching and learning in an important way.

For instance, students in the introductory mathematics class evaluated mathematics as boring, repetitive, focusing on minor things, not creative, and so on. This kind of perception often resulted from their negative learning experience of mathematics and more interestingly, lack of resource that they can rely on. Most students wanted something creative in mathematics class. However, one can be creative in mathematical practice only when his/her practice is firmly grounded on the culture and history of the mathematics community. Beginning mathematics students had harder time to understand the significance of a mathematical theorem because of the lack of their knowledge about history of the mathematics community: For them, the theorem was singled out from the historical context and as a result became less meaningful.

It is difference in epistemological style that makes one feel “others” strange. However, it is confliction due to epistemological difference that initiates learning and makes the impact of learning more fundamental, that is, negotiation of worldview of a learner, when considering the broad meaning of indigenous epistemology. A student comes to mathematics class with a limited and often a differing kind of epistemological standpoint from that shared in the mathematics community. As s/he interacts with different kinds of mathematicians in class, particularly mathematics teachers who have already been socialized into the communal epistemology, s/he becomes to see the practice of mathematics from a different perspective, especially, the indigenous epistemology of the mathematics community and begins to change as a whole person. In this regard, mathematics education can be seen as a process in which a mathematics teacher, as an old-timer, invites a student into the world of a vast inheritance historically accumulated to experience a specific mode of thought and awareness and helps him/her get transformed according to the indigenous epistemology of the mathematics community. However, it is important to note that such process of transformation is neither unilateral nor passive. Specifically, when considering that a communal epistemology is a product of historical contingency, it can be said that it is only one of standpoints providing a
vision for future. Thus, a mathematics teacher’s support based on the awareness of such differences will be essential for successful learning in mathematics class. In this perspective, investigation of deep culture in mathematics class will provide a theoretical ground for the improvement.

REFERENCES


